OPTION–B

Paper : MAT–HE–5026

(Mechanics)

1. Answer the following questions : ***(any ten)*** 1×10=10

(i) If a system of coplanar forces is in equilibrium, then what is the algebraic sum of the moment of the forces about any point in the plane ?

(ii) What is the resultant of the like parallel forces $P_1, P_2, P_3, \ldots$ acting on a body ?

(iii) If a particle moves under the action of a conservative system of forces, then what is the sum of its KE and PE ?

(iv) Define limiting equilibrium.

(v) Define the centre of gravity of a body.

(vi) Under what conditions the effect of a couple is not altered if it is transformed to a parallel plane ?

(vii) Write down the radial and cross-radial components of velocities of a particle moving on a plane curve at any point (r, θ) on it.

(viii) What is the resultant of a couple and a force in the same plane ?

(ix) What is dynamical friction ?

(x) What do you mean by terminal velocity ?

(xi) Define coefficient of friction.

(xii) What is the position of the point of action of the resultant of two equal like parallel forces acting on a rigid body ?

(xiii) What is the whole effect of a couple acting on a body ?

(xiv) Define simple harmonic motion.

(xv) What is the centre of gravity of a triangular lamina ?

(xvi) Define limiting friction.

(xvii) State the principle of conservation of energy.

(xviii) A particle moves on a straight line towards a fixed point O with an acceleration proportional to its distance from O. If x is the distance of the particle at time t from O, then write down its equation of motion.

2. Answer ***any five*** questions of the followoing : 2×5=10

(a) Write the laws of static friction.

(b) A particle moves in a circle of radius r with a speed v. Prove that its angular velocity is $\frac{v}{r}$.

(c) What are the general conditions of equilibrium of any system of coplanar forces ?

(d) The law of motion in a straight line is $s = \frac{1}{2}vt$. Prove that the acceleration is constant.

(e) Find the greatest and least resultant of two forces acting at a point whose magnitudes are P and Q respectively.

(f) Find the centre of gravity of an arc of a plane curve $y = f(x)$.

(g) State Hooke's law.

(h) Show that impulse of a force is equal to the momentum generated by the force in the given time.

(i) Write the expression for the component of velocity and acceleration along radial and cross radial direction in a motion of a particle in a plane curve.

(j) The speed v of a particle moving along x-axis is given by the relation $v^2 = n^2\left(8bx - x^2 - 12b^2\right)$. Prove that the motion is Simple Harmonic.

3. Answer ***any four*** questions of the following : 5×4=20

(a) The greatest and least resultants that two forces acting at a point can have magnitude P and Q respectively. Show that when they act at an angle α their resultant is $\sqrt{P^2\cos^2\frac{\alpha}{2} + Q^2\sin^2\frac{\alpha}{2}}$.

(b) I is the in centre of the triangle ABC. If three forces $\bar{P}, \bar{Q}, \bar{R}$ acting at I along $\overrightarrow{IA}, \overrightarrow{IB}, \overrightarrow{IC}$ are in equilibrium, prove that

$$\frac{P}{\sqrt{a(b+c-a)}} = \frac{Q}{\sqrt{b(c+a-b)}} = \frac{R}{\sqrt{c(a+b-c)}}$$

(c) Show that the resultant of three equal like parallel forces acting at the three vertices of a triangle passes through the centroid of the triangle.

(d) Prove that any system of coplanar forces acting on a rigid body can ultimately be reduced to a single force acting at any arbitrarily chosen point in the plane, together with a couple.

(e) Show that the sum of the Kinetic energy and Potential energy is constant throughout the motion when a particle of mass m falls from rest at a height h above ground.

(f) A point moves along a circle with constant speed. Find its angular velocity and acceleration about any point of the circle.

(g) Show that the work done against tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tension.

(h) A particle starts with velocity u and moves under retardation μ times of the distance. Show that the distance it travels before it comes to rest is $\frac{u}{\sqrt{\mu}}$.

4. Answer ***any four*** questions of the following : 10×4=40

(a) Forces P, Q and R act along the sides BC, CA and AB of a triangle ABC and forces P', Q' and R' act along OA, OB and OC, where O is the centre of the circumscribed circle, prove that

(i) $P\cos A + Q\cos B + R\cos C = 0$

(ii) $\frac{PP'}{a} + \frac{QQ'}{b} + \frac{RR'}{c} = 0$

(b) State and prove Lami's theorem. Forces P, Q and R acting along OA, OB and OC, where O is the circumcentre of triangle ABC, are in equilibrium. Show that

$$\frac{P}{a^2\left(b^2+c^2-a^2\right)} = \frac{Q}{b^2\left(c^2+a^2-b^2\right)} = \frac{R}{c^2\left(a^2+b^2-c^2\right)}$$

(c) (i) Find the centre of gravity of a uniform arc of the circle $x^2+y^2=a^2$ in the positive quadrant.

(ii) Find the centre of gravity of the arc of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant.

(d) A particle moves in a straight line under an attraction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Investigate the motion.

(e) A particle moves in a straight line OA starting from the rest at A and moving with an acceleration which is directed towards O and varies as the distance from O. Discuss the motion of the particle. Hence define Simple Harmonic Motion and time period of the motion.

(f) Find the component of acceleration of a point moving in a plane curve along the initial line and the radius vector. Also find the component of acceleration perpendicular to initial line and perpendicular to radius vector.

(g) A particle is falling under gravity in a medium whose resistance varies as the velocity. Find the distance and velocity at any time t. Also find the terminal velocity of the particle.

(h) The velocity component of a particle along and perpendicular to the radius vector from λr and $\mu\theta$. Find the path and show that radial and transverse component of acceleration are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\lambda + \frac{\mu}{r}\right)$.

(i) Find the component of acceleration of a point moving in a plane curve along the initial line and the radius vector. Also find the component of acceleration perpendicular to initial line and perpendicular to radius vector.

(j) A particle moves in a straight line under an attaction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Investigate the motion.

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